$\gamma \quad = 4(\sin\theta - \theta\cos\theta)/\theta_3$

$\theta \quad = kh = k(b-a)2N$

$h \quad = (b-a)2N$, where the interval (a,b) is divided into $2N$ subintervals of equal length.

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Experimental Study of Axial Flow in Wing Tip Vortices

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Introduction

THE vortex generated behind each wing tip of a fixed-wing aircraft may be hazardous to following aircraft,[1] whereas the vortex generated behind the tip of a helicopter rotor blade may interact with a following blade causing noise[2] and vibration.[3] A knowledge of the structure of a tip vortex is a necessary prerequesite to attempts to alleviate these problems.

The importance of axial flow in a tip vortex and of its interaction with the tangential flowfield has been investigated theoretically.[4,5] Experimental measurements in the wind tunnel[6-8] and in flight[9-11] have shown widely differing axial velocity distributions within the vortex core, with velocity excesses being found in some cases and deficits in others. This note describes a qualitative towing tank study of some of the factors which control the axial flowfield in a trailing vortex and attempts an explanation of the differing results of previous axial velocity measurements.

Experimental Details

The experiments were conducted in a small towing tank ($0.3 \times 0.3 \times 4$ m). Two rectangular wing models were used with NACA 6412 and NACA 0012 sections. Each wing had a span of 125 mm and a chord of 64 mm, and could be mounted, with its spanwise axis vertical, below the towing carriage. Most tests were carried out at Reynolds numbers of 3.4×10^4 and 6.8×10^4, based on wing chord.

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Three alternative tip configurations were tested for each wing. The basic tip was cut square, normal to the spanwise axis, and the rounded tip had a semicircular cross section at each chordwise station. A sharp edged tip was formed by extending the basic wing spanwise, using a thin brass plate cambered to the upper surface contour. The spanwis dimension of this extension plate was equal to the maximum wing thickness.

The hydrogen bubble technique was used for flow visualization. A horizontal cathode wire spanning the tank normal to the direction of wing motion generated a vertical curtain of bubbles between the wire and the water surface. As the wing passed through the curtain, axial flow was indicated by bubbles leaving the plane of the curtain. Bubbles moving in the same direction as the wing indicated a region of velocity deficit in a frame of reference fixed to the wing (as in a wind tunnel, for example), while bubbles moving in the opposite direction indicated a velocity excess in the wing fixed reference frame.

An alternative cathode consisted of a chordwise strip of aluminum foil 2 mm wide fixed to the wing upper surface at the tip. Bubbles generated by this cathode marked the core of the vortex behind the wing.

Results and Discussion

For all the wings tested, at angles of incidence of less than approximately 10°, the axial-flow pattern showed a velocity deficit on the core centerline at all stations behind the wing. At the core edges, a small velocity excess appeared at about 10-15 chord lengths behind the wing. The core edge excess persisted until approximately 30 chord lengths behind the wing before merging into an axial velocity deficit across the whole core.

At angles of incidence of 10-20°, the axial-flow patterns varied for the different wings. In the case of the square tipped NACA 6412 section wing an axial velocity excess appeared on the core centerline immediately behind the wing at an angle of incidence of approximately 10°. The magnitude of the centerline excess increased with increasing incidence. At some point downstream of the wing, the centerline velocity excess changed suddenly to a deficit. The changeover point was identifiable as the point at which bubbles from the bubble curtain which had been moving in the opposite direction to the wing suddenly reversed their motion (Fig. 1). The changeover point was also visible, using the wing tip cathode, as a region of sudden expansion of the vortex core, similar in appearance to a vortex burst. The changeover point moved closer to the wing with increasing incidence and decreased Reynolds number, as shown in Fig. 2.

The configuration of the wing tip affected the position of the axial-flow changeover point, as also shown in Fig. 2. Compared to the square tipped wing, rounding the tip edge shifted the point closer to the wing. For the sharp edged wing tip, the changeover occurred further from the wing at most angles of incidence.

Behind the NACA 0012 section wing, no velocity excess on the core centerline was detected at any incidence or for any tip configuration. Thus, the aerofoil section may be significant in determining the axial-flow distribution in the vortex core. Whether this effect is due to details of the tip flow for the different sections or to variations in the overall wing characteristics is not clear.

In the incidence range 10-20°, the subsequent flow development for all wing configurations was similar to that for angles of incidence below 10°, i.e., a small core edge excess appeared and later merged into a deficit across the whole core.

More than 30-35 chord lengths behind all the wings tested, at all angles of incidence, regions of axial velocity excess appeared at random along the core and moved erratically upstream and downstream. Each of these regions terminated in a sudden reversion to an axial velocity deficit, again similar in

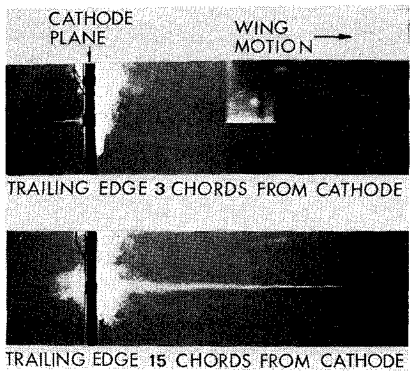

Fig 1 Axial flow patterns behind wing with square tip edge (NACA 6412 section; Reynolds number = 6.8×10^4; wing incidence = 12°).

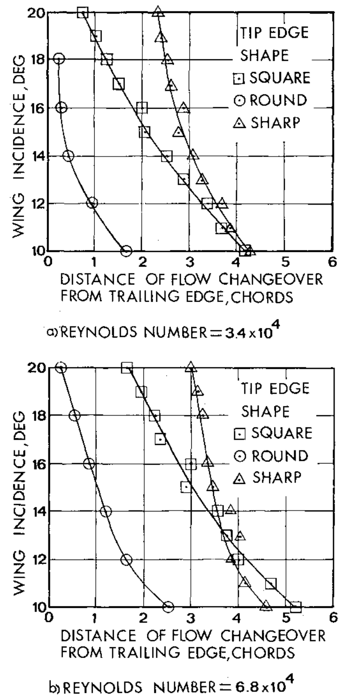

Fig 2 Position of point of changeover from axial velocity excess to axial velocity deficit behind NACA 6412 section wing.

appearance to a vortex burst. The disruption to the core by these random motions appeared to be the mechanism by which the vortex finally decayed.

Comparison with Other Experimental Results

Some of the flow features observed in the towing tank tests have been noted also in wind tunnel and other tests, using various wing sections and tip shapes, over a wide range of Reynolds numbers. For example, the tank tests have shown that, at certain stations behind a wing, an axial velocity deficit on the core centerline can change to a velocity excess as the wing incidence increases. Such a change has been observed in wind-tunnel tests by Chigier and Corsiglia.[6] The sudden change from an axial velocity excess to a velocity deficit on the core centerline has been observed in a wind-tunnel test by Scheiman, Megrail, and Shivers,[7] using smoke injected intermittently into a tip vortex. The flow distribution consisting of a centerline deficit and a core edge excess has been measured by Logan,[8] and is also apparent in towing tank photographs by Olsen.[12] Tombach[11] in flight experiments using smoke injected from a wing tip has noted a random motion along the core of apparent votex bursts, similar to the flow pattern seen in the decaying vortex in the present towing tank tests.

Conclusions

Towing tank tests have shown that the axial velocity distribution in a wing tip vortex can take the form of a centerline velocity excess, a centerline velocity deficit, or a centerline velocity combined with a core edge velocity excess. The velocity distribution at any station behind the wing depends on the wing section, tip shape, Reynolds number, wing incidence, and distance of the station from the wing. Only when these parameters are constant, can measured axial velocity distributions be compared. The shape of the streamwise tip edge appears to be particularly important. The published results of axial velocity measurements apply to a range of wing sections and tip shapes, and this may account for the variations in the axial velocity distributions.

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Analytical Solution for Inviscid Vortex Rollup from Elliptically Loaded Wings

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Introduction

THE structure of trailing vortices after rollup from aerodynamically loaded wings continues to be a subject of major interest to workers in the field of vortex research. It is desirable, for instance, to state, explicitly and analytically, the distributions of circulation and tangential velocity in the trailing vortex after rollup of the vorticity sheet that is

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